The relation between bulk and shear seismic quality factors

Igor B. Morozov

Abstract

Most seismic attenuation models show bulk dissipation in solids as significantly lower than dissipation in shear: $1/Q_K << 1/Q_\mu$. Here, we point out a remarkable consequence of such shear-dominated dissipation, which implies that materials behave "auxetically" with respect to mechanical-energy dissipation, *i.e*. analogously to elastic materials with negative Poisson's ratios. Although theoretically possible and existing in nature, auxetic materials are extremely rare among rocks. Thus, we seem to have an intriguing paradox of most of the Earth being such "exotic" in dissipation. To avoid such behavior, a lower bound of approximately $1/Q_K \geq 0.28/Q_\mu$ must be placed on bulk attenuation. However, this condition contradicts the approximation $1/Q_K \approx 0$ often used in seismic models. Three solutions to this problem are considered: 1) unlike elasticity, dissipation in Earth materials is indeed strongly shear-dominated, 2) the contradiction is due to an uncertainty of $1/Q_K$ in attenuation models, which could be adjusted, or 3) the fundamental complex-modulus model of dissipation needs to be reconsidered. Because the paradox principally arises from relying on the correspondence principle, solution (3) appears to be the most correct. Rigorous physical theories of dissipation in solids are formulated without the use of Q_K and Q_μ and consequently contain no such paradox. With either approach to this problem, symmetries and heterogeneity of stress fields need to be taken into account when constructing seismic-attenuation models.

1. Introduction

In most attenuation models of the Earth, the bulk dissipation factor, denoted Q_K^{-1}, is very low compared to the one for shear, Q_μ^{-1} (*e.g.*, PREM, Dziewonski and Anderson, 1981; Widmer *et al*., 1991; Romanowicz and Mitchell, 2007). For example, in PREM, the ratio $Q_K^{-1} \big/ Q_\mu^{-1}$ equals approximately 0.06 for the inner core and below 0.01 within the mantle. In consequence, most observations of seismic attenuation are predominantly explained by Q_μ^{-1} alone, which greatly simplifies the construction of attenuation models. The same assumption $Q_K^{-1} \ll Q_\mu^{-1}$ is also made in geodetic models (*e.g.*, Benjamin *et al*., 2006), which generally rely on seismological methodology for Earth's anelasticity (Efroimsky and Williams, 2009).

Taking the relation $Q_K^{-1} \ll Q_\mu^{-1}$ as either an observational or theoretical fact, the present note draws attention to a peculiar consequence of this relation. Materials with Q_K^{-1} below $\sim 0.28 Q_\mu^{-1}$ behave "auxetically" in respect to dissipation, which means that dissipation-related stresses within them are distributed similarly to elastic stresses in solids with negative Poisson's ratios. Elastic materials of such kind exist among minerals (Alderson and Evans, 2009) and likely their composites (Wei and Edwards, 1998); however, they are still extremely rare among rocks. Thus, we seem to have an interesting paradox of nearly the entire Earth showing an "exotic" behavior during internal friction. The paradox is amplified by the fact that the concepts of "specific dissipation parameters" Q_K^{-1} and Q_μ^{-1} are very general but nevertheless not well understood from the physical standpoint. These parameters are usually treated as fundamental constitutive properties of materials, and yet they arise not from considering the physics of inelastic deformation but from a mathematical extrapolation of the elastic case, known as the correspondence principle (Bland, 1960).

The correspondence principle and frequency-dependent Q_K^{-1} and Q_μ^{-1} comprise the well-known complex-modulus model of seismic anelasticity (Anderson and Archambeau, 1964; Cormier, 2011). Apart from relations $Q_K^{-1} \geq 0$ and $Q_\mu^{-1} \geq 0$, this model offers no rigorous constraints on the possible or likely values of these Q's. Anderson *et al*. (1965) suggested that "in general, losses in pure compression are smaller than losses in shear." This judgment likely referred to the perceived predominance of shear in frictional processes or to the estimates of $Q_K >$ 5000 from observations of the free-oscillation mode ${}_0S_0$ (Knopoff, 1964; for more on this, see *Discussion)*. However, compressibility is important for seismic phenomena, and therefore significant bulk friction can be expected. Thermoelastic effects are specifically sensitive to bulk deformation, and they are significant in rock deformation (Hayden *et al*., 1965). From similar arguments, it appears that Q_K^{-1} and Q_μ^{-1} *could* be comparable, and the relation between them should be structure-, mechanism-, and wave-dependent.

In keeping with the paradigm established in seismology, the argument of this paper is carried out on the basis of the complex-modulus model (Cormier, 2011). However, the conclusion ("auxetic" character of dissipation for $Q_K^{-1} \ll Q_\mu^{-1}$) casts serious doubts in this model itself. The reason for such anomalous behavior is much deeper than the uncertainty of the values of Q and relates to the fact that the mechanisms of mechanical-energy dissipation cannot be reduced to only two "quality factors" treated by the correspondence principle. Empirical quasi-static creep or phase-lag models used in materials science (*e.g.*, Nowick and Berry, 1972; Karato, 2008) should not be directly extrapolated to seismic waves. The dynamics of solids and fluids are similar and best described by the language of analytical continuum mechanics and thermodynamics (Landau and Lifshitz, 1986; Müller *et al*., 2010; Morozov, 2010). These approaches recognize numerous physical factors of energy dissipation but neither the correspondence principle nor "material *Q*." These approaches are also free from the surprising "paradox" discussed here.

The concept of material *Q* was proposed by generalizing the apparent *Q* values measured in various experiments. Body and surface waves, creep experiments (Lomnitz, 1957), free oscillations of the Earth or forced oscillations of specimens in the lab (*e.g*., Jackson and Paterson, 1993) lead to different apparent *Q*'s, from which the respective *in situ* *Q*'s are inferred. In particular, the bulk Q_K^{-1} is derived from body-wave Q_P and Q_S factors by using the relation between the bulk, shear, and P-wave *elastic moduli* (*K*, μ, and *M*, respectively): $K = M - 4\mu/3$. It is assumed that the same relation holds for the imaginary parts of these moduli, yielding (Anderson and Archambeau, 1964; Knopoff, 1964):

$$Q_\mu^{-1} \equiv Q_S^{-1}\text{, and } KQ_K^{-1} \equiv MQ_P^{-1} - \frac{4}{3}\mu Q_S^{-1}, \tag{1}$$

where $\operatorname{Im} K \equiv -KQ_K^{-1}$ and similarly for other moduli. Note that these relations utilize no analysis of energy dissipation but arise purely from the correspondence principle, *i.e*. assumption that *K*, μ, and *M* represent valid complex quantities. It is therefore important to see what (potentially unexpected) physical consequences this assumption may entail. The most direct of such consequences is on the symmetry of the frictional stress field.

As shown in sections 2 and 3, despite the perceived analogy with the elastic case, this analogy breaks down on the observed values of *Q*-factors. Seismic models often suggest low Q_K^{-1}, which means that the *Q*-counterpart of the first Lamé modulus, λ, is negative: $Q_\lambda^{-1} < 0$. This leads to the character of the resulting viscous-stress field which is very different from that of the elastic stress and can be called "auxetic". To restore the qualitative analogy, Q_K^{-1} must satisfy a *lower bound* (section 4):

$$Q_K^{-1} \geq \frac{2\mu}{3K} Q_\mu^{-1}. \tag{2}$$

This relation is merely an analog to the well-known relations $\lambda \geq 0$ and $K \geq 2\mu/3$ for elastic constants. Further (section 5), if looking for a simple assumption about bulk attenuation which could be made in the absence of relevant data, a physically plausible choice suggests $Q_K^{-1} \approx 2\mu Q_\mu^{-1}/3K$, instead of the traditional $Q_K^{-1} \approx 0$. With such relation between the two *Q*'s, the internal friction behaves in a simple way, with frictional stress tensor being proportional to the strain-rate tensor. Unfortunately, this choice appears to conflict with many of the existing attenuation models, and so we have to either: 1) revisit and adjust some of these models, 2) look for reasons for the general shear-dominated character of internal friction, or 3) reconsider the underlying model (1) and the correspondence principle. Our argument is strongly in favor of the last of these propositions.

2. Analogies between viscous and viscoelastic stresses

The following explanation of the constraint (2) on bulk attenuation was suggested by one of the anonymous reviewers of this manuscript. Consider the general goal of constructing a theory of mechanical-energy dissipation in a solid (Ben Menahem and Singh, 1981; p. 848). In the viscoelastic approach, the solution is based on two ideas: 1) borrowing viscosity (strain-rate to stress) relations from fluid mechanics, and 2) including this viscosity into *strain-stress* relations for harmonic oscillations by using the correspondence principle. Viscosity is therefore represented by the imaginary part of the corresponding elastic modulus (*e.g.*, Young's, P-wave, shear, or bulk), as appropriate for the oscillation considered. The key question is what types of viscosities to expect in a solid. The analogy with fluid mechanics suggests Stokes' viscosity (*ibid*); however, the classical Stokes' fluid is usually viewed as incompressible and offers no guidance for bulk dissipation. By extending this model to general linear bulk dissipation, we obtain the Newtonian fluid, in which the stress tensor is:

$$\sigma_{ij} = \left(-p + \lambda' \frac{\partial u_k}{\partial x_k} \right) \delta_{ij} + \mu' \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \tag{3}$$

where p is the pressure, $\mathbf{u}$ is the velocity, and μ' and λ' and are "dynamic" and "second" viscosities, respectively. The imaginary parts of the viscoelastic moduli in (1) are related to these parameters as $\operatorname{Im} \lambda = -\omega\lambda'$ and $\operatorname{Im} \mu = -\omega\mu'$, where ω is the frequency of oscillation.

Similarly to elasticity, in order to guarantee non-negative energy dissipation, both μ' and the bulk viscosity $K' = \lambda' + 2\mu'/3$ must be non-negative. Consequently, similarly to λ, λ' can in principle be negative. However, λ is non-negative for a vast majority of natural materials, and therefore we can expect that λ' should commonly be non-negative as well. This gives the constraint (2). Moreover, the elastic λ is usually close to μ within the deep Earth, and we can conjecture that the imaginary parts of these moduli might also be comparable. This suggests a range of "likely" K' values from $\sim 2\mu'/3$ to $\sim 5\mu'/3$ for the Earth. A detailed mechanical explanation of these constraints is given in the following section.

3. Symmetries of the elastic and frictional stress fields

In a propagating wave, the deformation is constrained, and "auxetic" effects are manifested by the properties of the resulting stress fields. To understand the stress fields, it is again useful to look at the elastic case first. All properties of the elastic stress in a deformed solid arise from the expression for its elastic-energy density, E_{el}. For an isotropic medium, E_{el} can be written as a function of either the total strain tensor, ε_{ij}, or of its dilatational and deviatoric ("pure shear") parts, $\Delta \equiv \varepsilon_{kk}$ and $\tilde{\varepsilon}_{ij} = \varepsilon_{ij} - \Delta\delta_{ij}/3$, respectively (Landau and Lifshitz, 1986):

$$E_{el} = \frac{\lambda}{2}\Delta^2 + \mu\varepsilon_{ij}\varepsilon_{ij} \equiv \frac{K}{2}\Delta^2 + \mu\tilde{\varepsilon}_{ij}\tilde{\varepsilon}_{ij}. \tag{4}$$

Note that λ and K are the isothermal moduli. For adiabatic deformations, these moduli are higher and E_{el} becomes the free energy (*ibid*).

From thermodynamics, the key requirement is the stability of equilibrium, which requires $E_{el} \geq 0$ for any deformation. Deformations with zero shear ($\tilde{\varepsilon}_{ij} = 0$) and pure shear ($\Delta = 0$) can be implemented independently, and therefore we have $\mu \geq 0$ and $K \geq 0$ in (4), and consequently $\lambda \geq -2\mu/3$. These are the only absolute constraints on the values of elastic moduli.

Depending on the sign of λ, materials behave differently upon compression or extension (Figure 1). In almost all natural materials, $\lambda \geq 0$, so that when stretched along one dimension, bodies contract laterally (Figure 1a). The (Poisson's) ratio of the relative transverse shrinkage to the extension is non-negative: $\nu = \lambda / \left[2\left(\lambda + \mu \right) \right] \geq 0$. In a P wave, the deformation is strictly uniaxial, and the Poisson's ratio becomes the ratio of the principal stresses present within the wave. For a strain in the direction of axis X, the axial strain equals $\varepsilon_{11} = \Delta$, all other $\varepsilon_{ij} = 0$, and therefore:

$$\boldsymbol{\sigma} = \begin{pmatrix} \left(\lambda + 2\mu\right)\Delta & 0 & 0 \\ 0 & \lambda\Delta & 0 \\ 0 & 0 & \lambda\Delta \end{pmatrix} \equiv \begin{pmatrix} \left(K + \frac{4\mu}{3}\right)\Delta & 0 & 0 \\ 0 & \left(K - \frac{2\mu}{3}\right)\Delta & 0 \\ 0 & 0 & \left(K - \frac{2\mu}{3}\right)\Delta \end{pmatrix}. \tag{5}$$

Thus, for $\lambda > 0$ (*i.e.*, $K > 2\mu/3$), all stresses are of the same sign as Δ, and the ratio of lateral to longitudinal stresses equals ν (Figure 1b). For $\lambda < 0$ ($K < 2\mu/3$), this ratio is negative, which means that when stretched, such material tends to expand transversely (Figure 1c). For the intermediate case of $\lambda = 0$, the material stretches as a group of independent ribbons, with zero stresses in the transverse direction (Figure 1d).

For stresses caused by internal friction within an anelastic material, similar relations take place. In the linear viscoelastic model, dissipation is included in the imaginary parts of the elastic moduli; for example (see also eq. (1)):

$$K \to \tilde{K} \equiv K - iKQ_K^{-1}. \tag{6}$$

Note that eq. (3) then gives $Q_K^{-1} = \omega K' / K$, which is the only frequency dependence allowed by the Newtonian-fluid analogy. This frequency dependence is found inadequate (Knopoff, 1964) and replaced with arbitrary $Q_K^{-1}\left(\omega\right)$. Phenomenologically, energy dissipation can also be characterized by strain-stress phase lags, for example: $\tan\phi_\mu = -\operatorname{Im}\tilde{\mu} / \operatorname{Re}\tilde{\mu} \equiv Q_\mu^{-1}$. Such phase lags are often treated as material properties and used for lab measurements of Q^{-1} at seismic frequencies (*e.g.*, Jackson and Paterson, 1993; Lakes, 2009). In the time domain, complex $\tilde{K}$ and $\tilde{\mu}$ lead to time-dependent strain-stress relations, such as the creep function, "material memory," or "compliance."

Mechanical-energy dissipation is always produced by internal forces directed against the velocities of the medium. Therefore, in a harmonic P wave, frictional stress equals $\boldsymbol{\sigma}_D = i\tilde{\boldsymbol{\sigma}}_D \exp(-i\omega t)$, where:

$$\tilde{\boldsymbol{\sigma}}_D = -\begin{pmatrix} \left(\left|\operatorname{Im}\tilde{K}\right| + \frac{4}{3}\left|\operatorname{Im}\tilde{\mu}\right|\right)\Delta & 0 & 0 \\ 0 & \left(\left|\operatorname{Im}\tilde{K}\right| - \frac{2}{3}\left|\operatorname{Im}\tilde{\mu}\right|\right)\Delta & 0 \\ 0 & 0 & \left(\left|\operatorname{Im}\tilde{K}\right| - \frac{2}{3}\left|\operatorname{Im}\tilde{\mu}\right|\right)\Delta \end{pmatrix}, \qquad (7)$$

because both $\operatorname{Im}\tilde{K}$ and $\operatorname{Im}\tilde{\mu}$ are non-positive. As in auxetic elastic materials, when $\left|\operatorname{Im}\tilde{K}\right| < 2\left|\operatorname{Im}\tilde{\mu}\right|/3$, the forces of friction are strongly anisotropic: positive along axis X and negative along Y and Z (Figure 1c). If we consider a specimen of such material with free boundaries on the sides and apply a uniform axial extension to it, $\dot{\varepsilon}_{xx} > 0$ (Figure 1a), then viscous forces will also tend to stretch it transversely (*i.e.*, reduce the Poisson's shrinkage). For $\dot{\varepsilon}_{xx} < 0$, the specimen will shrink additionally in the transverse direction. Such behavior can be described as "auxetic" in dissipation.

The above anisotropic pattern of frictional forces arises from the predominance of shear over bulk dissipation. To prevent such effects, we need to require a lower bound on $\left|\operatorname{Im}\tilde{K}\right|$ in "ordinary" materials:

$$\left|\operatorname{Im}\tilde{K}\right| \geq \frac{2}{3}\left|\operatorname{Im}\tilde{\mu}\right|. \qquad (8)$$

This constraint simply means that $\operatorname{Im}\tilde{\lambda} < 0$, which is analogous to $\lambda \geq 0$ and non-negative Poisson's ratios in natural materials.

4. Implications for seismic Q

The above constraint (8) shows that to avoid "auxetic" (anisotropic) frictional stresses within a material, its bulk dissipation should be comparable to the shear one. This observation has important implications for the existing models of seismic *Q*. By inferring values of Q_K and Q_μ from the correspondence principle (1), constraint (8) leads to (2). For typical P- and S-wave velocity ratios of $V_P/V_S = \sqrt{3}$ (Poisson's solid with $\nu = 0.25$), this gives approximately $Q_K^{-1} \geq 0.28 Q_\mu^{-1}$.

Constraint (2) is contradicted by many current interpretations of nearly "pure shear" dissipation, $Q_K^{-1} \approx 0$. The meaning of this approximation can again be illustrated by an inverse analogy with elasticity. Assume that similarly to "shear-dominated" dissipation, we wish to construct a medium with "shear-dominated" elasticity. At the first glance, taking $K = 0$ ("pure shear") might appear to be the easiest approach. However, such a medium would also have $\lambda = -2\mu/3$ and a negative Poisson's ratio (Figure 1c). Taking $\lambda = 0$ (or even $0 \leq \lambda < \mu$) instead makes a more reasonable alternative. The dominance of shear is still achieved ($K < \mu$), and both of these media behave identically in S waves (in which $\Delta = 0$), but the one with $K = 0$ develops auxetic stresses in P waves. Thus, because elastic "pure shear" is extremely rare in natural materials, we suggest that "dissipation pure share" might be rare too.

5. A simplifying assumption for mechanical-energy dissipation

Relation $Q_K^{-1} = 0$ is often used as a simplifying assumption about bulk attenuation in the absence of data for its measurement (*e.g.*, Dziewonski and Anderson, 1981). However, as shown above, this approximation implies properties of frictional stresses which are contrary to those which might be expected from the correspondence-principle analogy. To overcome this difficulty, an alternate simplification can be made by replacing the inequalities (8) and (2) with equalities. This approximation has a simple physical meaning. With $\operatorname{Im}\tilde{K} = 2\operatorname{Im}\tilde{\mu}/3$, $\operatorname{Im}\tilde{\lambda} = 0$, and the force of friction applied to an elementary surface *dS* oriented with a normal vector **n** is proportional to the strain rate tensor, $\dot{\varepsilon}_{ij}$:

$$f_i = -2\operatorname{Im}\tilde{\mu}\dot{\varepsilon}_{ij}n_j dS \ . \tag{9}$$

By selecting, for example, vector **n** in the direction of axis *Z*, we can see that such a force is always directed against the corresponding strain rate, and the medium experiences no “sideways” friction (Figure 2). Such behavior might be the easiest to expect without knowledge of the specific mechanism of friction.

If $\operatorname{Im}\tilde{K} = 2\operatorname{Im}\tilde{\mu}/3$, then the corresponding relation for the *Q*-factors is $Q_K^{-1} \approx \left(2\mu/3K\right)Q_\mu^{-1}$ (see relation (2)). For plane P- and S waves in a Poisson’s solid, this gives $Q_P \approx 1.5Q_S$ (eq. (1)). This relation is close to seismological observations, although with notable exceptions (Knopoff, 1964). However, importantly, this is an *upper* bound on Q_P. This bound is not satisfied in global attenuation models, in which $Q_P \approx 1.8Q_S$ ($Q_K^{-1} \approx 0$ in eq. (1)).

6. Discussion and conclusions

The point of the present paper is very simple: because the complex-modulus model is derived from an analogy with elasticity, it seems “unnatural” or “undesirable” that the predominance of bulk elasticity ($\lambda > 0$, and commonly even $\lambda \sim \mu$) is replaced with a sharply opposite relation $\lambda' \approx -2/3\mu'$ for practically all Earth materials. In terms of the resulting stress fields, this means that frictional stresses are always strongly anisotropic (shear-dominated). In the elastic domain, such behavior is only found in materials with negative Poisson’s ratios, which are extremely rare in nature.

Although uncomfortable from the analogy with elasticity, pervasive $\lambda' < 0$ is nevertheless present in many attenuation models of the Earth (*e.g.*, Dziewonski and Anderson, 1981). This is the paradox of “auxetic” character of seismic attenuation mentioned in the Introduction. Some authors (*e.g.*, Widmer *et al.*, 1991) indicate significant trade-offs between Q_K^{-1} and Q_μ^{-1}, which could possibly be used for honoring the constraint (2) and avoiding this problem. However, the simplicity and generality of this contradiction suggests that its cause is likely beyond the numerical uncertainties in *Q* models.

Our explanation of the above paradox is that it rests principally with the complex-modulus model of seismic attenuation. Despite what is often thought, parameters Q_K^{-1} and Q_μ^{-1} are not real physical properties but only attributes defined so that through expressions (1), they reproduce the quality factors *for body P and S waves in uniform media*. These attributes are not guaranteed to predict dissipation in other cases such as, for example, for surface waves in layered media (Morozov, 2012). The correspondence principle relates dissipation specifically to the elastic energy, for example $\dot{\bar{E}}_K = -\omega \bar{E}_K Q_K^{-1}$ (where E_K is the bulk energy given by the first term in the right-hand side of (4)), and assumes Q_K^{-1} to be independent of the wave type. However, in reality, dissipation is not directly related to the elastic energy (see section 2). In other types of waves, such as Love and Rayleigh surface waves, the values of $Q_K^{-1} = -\dot{\bar{E}}_K \big/ \omega \bar{E}_K$ and similarly Q_μ^{-1} may be different. Therefore, there should hardly exist any definite relation between these quantities. In particular, we can hardly expect $Q_K^{-1} \ll Q_\mu^{-1}$ or *vice versa*, because this limit would eliminate one of the already scarce degrees of freedom in the model of energy dissipation.

Knopoff (1964) made several insightful remarks on the above subject. He noted that low bulk dissipation was principally suggested by the very high $Q \approx 5700$ of the radial free-oscillation mode ${}_0S_0$ ($1/Q \approx 0.175 \cdot 10^{-3}$; REM, 2010), which suggests roughly $Q_K \approx 600$. At the same time, such high Q implies $Q_P > Q_S$, which often disagrees with observations. For example, from surface-wave Q's at periods $T < 300$ s, Knopoff (1964) estimated $Q_P \approx 25$ and $Q_S \approx 110$, which was far in the opposite relation. He concluded that $Q_P < Q_S$ was consistent with observations, and the high Q of ${}_0S_0$ may be a fortuitous consequence of the viscoelastic model (1). If partial melting is present in the upper mantle, relations (1) may be inappropriate (*ibid*). We also note that as mentioned in the preceding paragraph, the value of Q_K inferred from mode ${}_0S_0$ should likely be different from Q_K inferred from body waves by using eq. (1).

The correspondence principle (CP) (eqs. (1) and (6)) deserves a separate comment, because it is critical for modeling, inversion, and even measurement of Q (Jackson and Paterson, 1993; Romanowicz and Mitchell, 2007). There are several specific indications that this principle does not work in heterogeneous media, and so its utility may be very limited. For example, Lines *et al.* (2008) modeled seismic reflections from contrasts in Q, and D. Aldridge (personal communication) pointed out that polarities of these reflections were opposite to those predicted from the CP. Morozov (2011) considered this example in detail and showed that the problem arose from mixing the notions of phase velocity and wave speed within the medium. The CP only rigorously applies to the phase velocity, $V_{\text{phase}} = \omega / k$ (where k is the wavenumber), whereas the wave speed is a combination of the elastic moduli and density. In a heterogeneous medium, these quantities are different, and attributing imaginary parts to the wave speeds contradicts boundary conditions in wave mechanics. Likely because of this general reason, the CP-based approach encounters significant internal difficulties even in basic plane-wave reflection problems (Krebes and Daley, 2007).

Thus, it appears that the approach explaining mechanical-energy dissipation within the Earth by a "material Q" may need to be critically reviewed. Only two Q-type parameters should generally be insufficient for describing the broad variety of dissipation processes, and their association with elastic moduli is not warranted by mechanics. Energy dissipation cannot be so similar to elasticity as presented by eqs. (1). Because of this conceptual frugality, frequency dependence of Q is commonly used as a proxy for physical mechanisms of dissipation. However, once allowing arbitrary $Q(\omega)$, the complex-modulus models becomes uninformative, over-parameterized and affected by variations of geometric spreading and other model assumptions (Morozov, 2008, 2010). Pairing attenuation properties with elastic moduli also makes these models inapplicable to fluids, in which $\mu = 0$. This makes the descriptions of solid-fluid systems, such as terrestrial planets, overly complex.

With typical $Q \approx 100$–1000, the effects of seismic-wave attenuation are weak compared to those of elastic stresses, geometric spreading, ambient pressure, heterogeneity and temperature. Consequently, no spectacular anomalies such as "auxetic" lateral deformations of the medium in a seismic wave can likely be expected from the various assumptions about the Q's. Nevertheless, subtlety of the effects does not mean that their theory can be constructed arbitrarily. Quite oppositely, the principles of physics, such as locality of interactions, causality, symmetries and thermodynamics become all the more important in this case. Energy-dissipation effects are measurable, provide valuable links to the physical state of the Earth's interior, and they need to be interpreted carefully.

In conclusion, the described (potentially?) anomalous behavior of the Earth's medium in internal friction could be only one indication of the difficulties of the correspondence-principle based approach to seismic attenuation. On the other hand, rigorous physical approaches to energy dissipation in fluids and solids have been known for some time (*e.g.*, Biot, 1956; Landau and Lifshitz, 1986). Nevertheless, significant work is still needed in order to incorporate them in global models of seismic-wave attenuation. Such rigorous models should likely be formulated without the use of Q_K^{-1} and/or Q_μ^{-1}, which will remove the above anomaly.

Data and Resources

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Figures

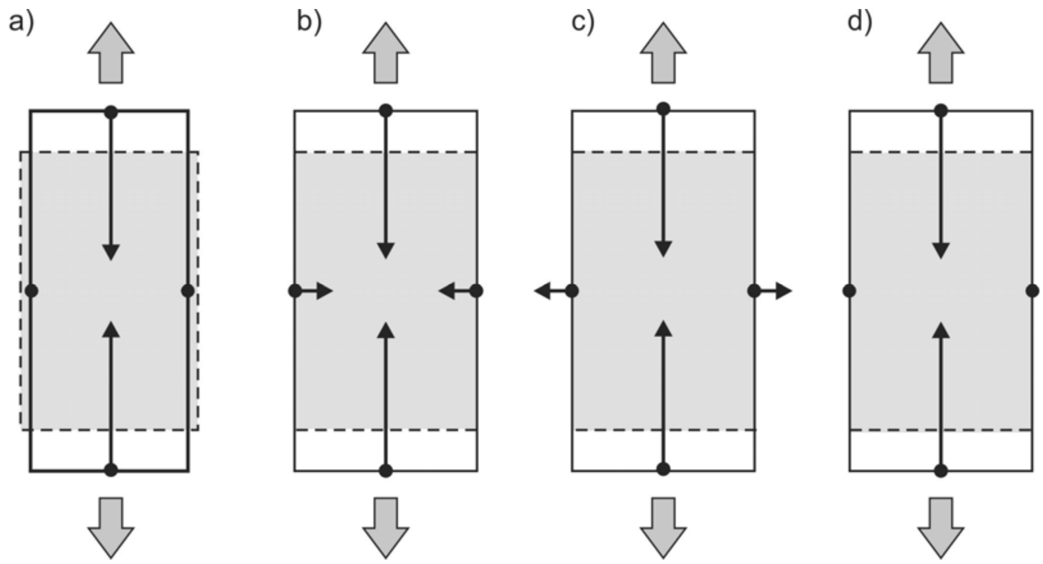

Figure 1. Behavior of materials under axial stretching: a) ordinary material with $\lambda > 0$ with free boundaries in transverse directions (example of Poisson's solid with $\nu = 0.25$ shown), b) the same body constrained to strictly axial deformation, as in a P wave, c) same deformation, auxetic material with $\nu = -0.25$, c) neutral ($\nu = 0$). Gray rectangles show the undeformed specimen (the same in all cases). Black arrows and dots indicate the stress responses within the material.

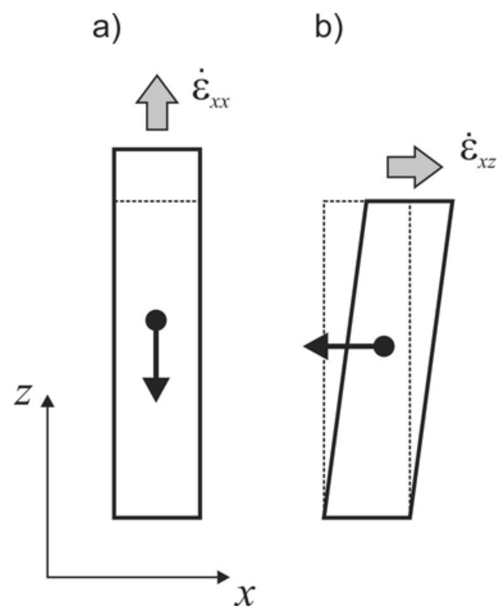

Figure 2. Viscous forces in eq. (9) (black arrows) applied to a cross-section of the body along z = const: a) for extension along axis Z, b) for shearing. Note that the forces are always directed against deformation rates (gray arrows labeled with non-zero components of the strain-rate tensor, $\dot{\varepsilon}_{ij}$).